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FIRST SEMI-ANNUAL STATUS REPORT

on

✓ NASA-Langley Grant No. NAG-1-250

for

Period Covering 18 January 1982 through 16 July 1982.

"DESIGN OF HELICOPTER ROTOR BLADES FOR OPTIMUM DYNAMIC CHARACTERISTICS"

by

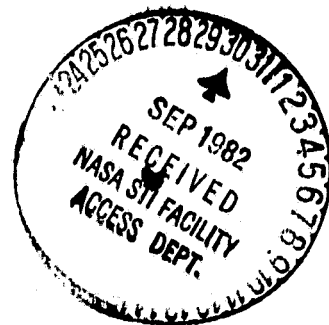
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September 15, 1982



(NASA-CR-169352) DESIGN OF HELICOPTER ROTOR
BLADES FOR OPTIMUM DYNAMIC CHARACTERISTICS
Semiannual Status Report, 18 Jan. - 16 Jul.
1982 (Washington Univ.) 33 p HC A03/MF A01

N82-33374

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CSCI 01C G3/05

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1. INTRODUCTION

1.1 Rotor Blade Design

The design of helicopter rotor blades involves not only considerations of strength, survivability, fatigue, and cost, but also requires that blade natural frequencies be significantly separated from the fundamental aerodynamic forcing frequencies (e.g. Ref. 1). A proper placement of blade frequencies is a difficult task for several reasons. First, there are many forcing frequencies (at all integer-multiples of the rotor RPM) which occur at rather closely-spaced intervals. For example, 5/rev and 6/rev are less than 20% apart. Second, the rotor RPM may vary over a significant range through the flight envelope, thus reducing even further the area of acceptable natural frequencies. Third, the natural modes of the rotor blade are often coupled because of pitch angle, blade twist, offset between the mass center and elastic axis, and large aerodynamic damping. These couplings complicate the calculation of natural frequencies. In fact, the dependence on pitch angle makes frequencies a function of loading condition, since loading affects collective pitch. Fourth, the centrifugal stiffness often dominates the lower modes, making it difficult to alter frequencies by simple changes in stiffness.

In the early stages of the development of the helicopter, it was believed that helicopter vibrations could be reduced (and even eliminated) by the correct choice of structural coupling and mass stiffness distributions. However, it is easy to imagine how difficult it is to find just the proper parameters such that the desired natural frequencies can be obtained. The difficulties in placement of natural frequencies have led, in many cases, to preliminary designs which ignore frequency placement. Then, after the structure is

"finalized" (either on paper or in a prototype blade), the frequencies are calculated (or measured) and final adjustments made. Reference (2) describes the development of the XH-17 helicopter in which a 300-lb weight was added to each blade in order to change the spanwise and chordwise mass distribution and thereby move the first flapwise frequency away from 3/rev. However, these types of alterations are detrimental to blade weight, aircraft development time, and blade cost. In addition, corrections usually are not satisfactory, and the helicopter is often left with a noticeable vibration problem.

The state-of-the-art in helicopter technology is now to the point, however, that it should be possible to correctly place rotor frequencies during preliminary design stages. There are several reasons for this. First, helicopter rotor blades for both main rotors and tail rotors are now being fabricated from composite materials (Refs. 3 and 4). This implies that the designer can choose, with limited restrictions, the exact EI distribution desired. Furthermore, the lightness of composite blades for the main rotor usually necessitates the addition of weight to give sufficient autorotational blade inertia. Thus, there is a considerable amount of flexibility as to how this weight may be distributed. Second, the methods of structural optimization and parameter identification are now refined to the point where they can be efficiently applied to the blade structure. Some elementary techniques have already been used for the design of rotor fuselages (Ref. 5). It follows that the time is right for the use of structural optimization in helicopter blade design. Some work on this is already under development, and, although not published, some companies are already experimenting with the optimum way to add weight to an existing blade in order to improve vibrations.

The purpose of the work discussed here is to investigate the possibilities (as well as the limitations) of tailoring blade mass and stiffness distributions to give an optimum blade design in terms of weight, inertia, and dynamic characteristics.

The major objectives of the work are:

- 1) To determine to what extent changes in mass or stiffness distribution can be used to place rotor frequencies at desired locations.
- 2) To establish theoretical limits to the amount of frequency shift.
- 3) To formulate realistic constraints on blade properties based on weight, mass moment of inertia, size, strength, and stability.
- 4) To determine to what extent the hub loads can be minimized by proper choice of EI distribution.
- 5) To determine if the design for minimum hub loads can be approximated by a design for a given set of natural frequencies.
- 6) To determine to what extent aerodynamic couplings might affect the optimum blade design.
- 7) To determine the relative effectiveness of mass and stiffness distribution on the optimization procedure.
- 8) To determine to what extent an existing blade could be optimized with minimal changes in blade structure.
- 9) To develop several "optimum profiles" for rotor blades operating under various standard conditions.

The work is to focus on configurations that are simple enough to yield clear, fundamental insights into the structural mechanisms but which are sufficiently complex to result in a realistic result for an optimum rotor blade.

1.2 Overview of Optimal Structural Design

Most approaches to optimal structural design may be classified into three categories. (For recent review articles see Refs. 6 and 7.) One such category is "variational methods." These generally rely on techniques from the mathematical theory of the calculus of variations, and, when applicable, often provide useful physical insight into the nature of an optimal design. Unfortunately, only relatively simple problems can be solved by this approach, since the mathematics becomes intractable when complex engineering structures are considered.

A second category of structural optimization techniques consists of the application of mathematical programming methods together with the discretization of the structure by finite element techniques. This approach to optimization was founded in 1960 (Ref. 8) with the hope that more complex structures could be analyzed than were possible when using the analytical techniques of the calculus of variations. However, in the late 60's it became apparent that mathematical programming methods had limitations of their own, namely, unacceptably long computation times occurring when the number of design variables become large (over 20-100, depending on the type of structure). Fortunately, several improvements developed over the last few years appear to have significantly extended the capability of the mathematical programming approach, and, as a result, it is this approach we intend to draw upon for solution techniques in the proposed research. In a later section of this proposal, after our design problem has been formulated, we will discuss these recent improvements in the approach.

A third category of structural optimization approaches is the "optimality criterion" approach in which an equation expressing some necessary condition of optimality is used as the basis for constructing an iterative (successive

re-design) procedure. Originally developed because of dissatisfaction with mathematical programming techniques, the optimality criterion approach initially relied on intuitive optimality criteria such as constant stress-ratio and uniform strain-energy density conditions. More recently, optimality criteria (and associated re-design equations) have been derived from the Kuhn-Tucker conditions (see, e.g., Ref. 9) for a constrained minimization problem.

The optimality criterion approach seems especially well-suited to problems with a large number of design variables. Since our proposed design problems will have a moderate number of variables and since deriving efficient re-design equations for our problems is not immediately straightforward, we initially prefer the mathematical programming approach over the optimality-criterion approach.

A structural optimization computer program, called CONMIN, is available from NASA. It is this program that is used in our present work. CONMIN is based on the mathematical nonlinear programming method of feasible directions.

Nevertheless, if CONMIN proves to be too expensive computationally (or in any other way unsuccessful for our work) we may turn to other approaches.

2. BACKGROUND

2.1 Formulation of Problem

Because numerically-based optimization is best carried out with discrete variables, the finite element technique stands as the most logical choice for the blade model. A recent research project (Ref. 10) has resulted in a finite-element computer program that is ideally suited to the work proposed here. The program allows for tapered, twisted finite elements in a rotating environment. The existing code can calculate natural frequencies, (with or without aerodynamic terms) and forced response.

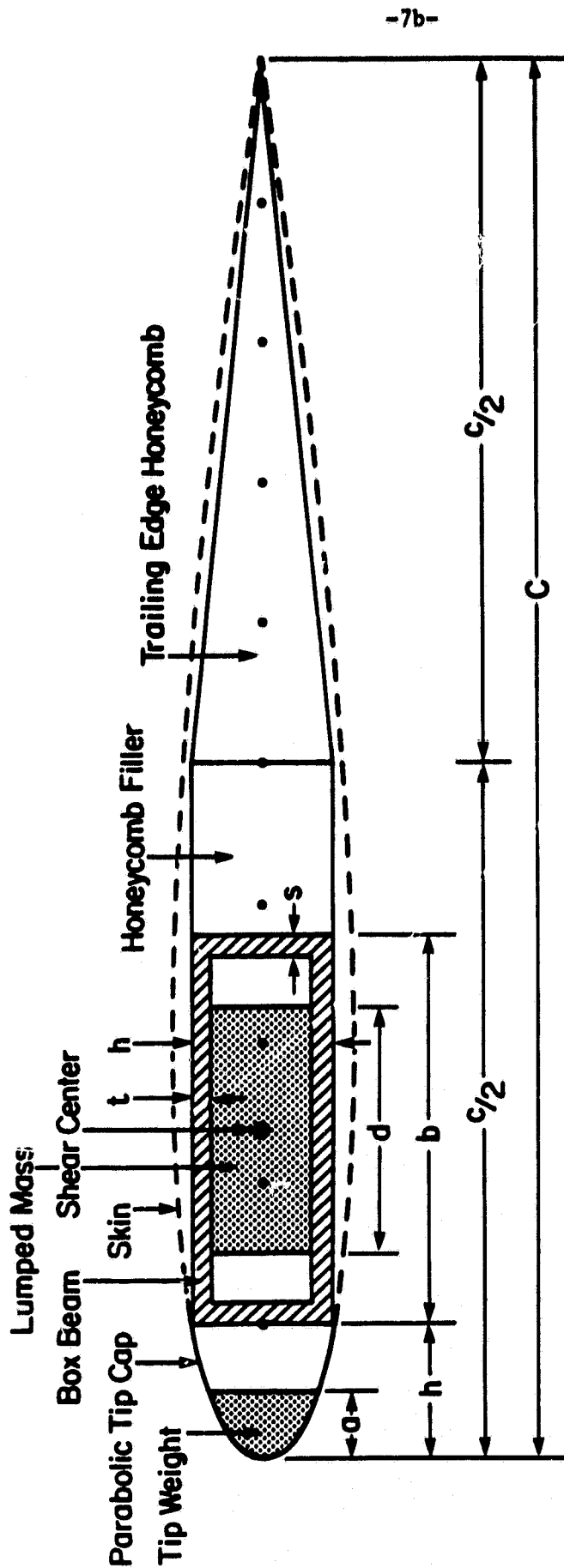
Another important aspect of the rotor blade optimization problem is the selection of the optimality criteria and constraints to be imposed. Our design problem has certain features which are unusual compared to typical problems occurring in the structural optimization literature. There are basically three categories of criteria. In the first class, one would minimize weight given constraints on the natural frequencies (i.e. frequency "windows"). In this case, a constraint on rotary inertia is also implied since a rotor must have sufficient inertia to autorotate. The advantage of this approach is that it is directly related to the physical realities of design. The disadvantage, however, is that the first guess will probably not be feasible (that is will not have frequencies that fall in the "windows"). This can be a stumbling block to convergence. A second type of criteria is one in which the objective is to minimize the discrepancies between desired frequencies and actual frequencies. The constraint then becomes a window on autorotational inertia. Although this avoids unfeasible solutions, it does not directly minimize weight (although weight is limited by the autorotational constraint). An objective function could be constructed that combined mass and frequency

placement, but the relative weightings of the two components is not obvious. The third category of constraint is to minimize vibrations directly without regard to frequency placement. Although this appears on the surface to be the perfect solution, there are problems. First, calculation of vibrations is an order-of-magnitude more difficult than the calculation of frequencies. Second, past efforts at this have resulted in strange designs, incompatible with standard helicopter practice. Third, there is still the problem of the weight-vibration trade-off. In this work, we intend to concentrate on the first two categories with some attention to the third.

Another type of constraint involved in the problem is the limitation on structural properties. The blade planform, airfoil, and twist are chosen by the aerodynamicist on the basis of performance. The structural engineer must choose his design to fit in the aerodynamic envelope given. There are five structural parameters to be chosen: 1) flapping stiffness, 2) inplane stiffness, 3) torsional stiffness, 4) mass, and 5) torsional moment of inertia. In practice, these cannot be chosen completely independently. Figure 1 shows the envelope of a typical blade section. All stiffness is assumed to reside in a box-beam of dimension $b \times h$ with thicknesses s and t . This beam is placed as far forward as possible (to keep the elastic axis near the $1/4$ chord). Mass properties are due to the box-beam, skin, honeycomb, and two lumped masses. The lumped mass in the tip is typical of rotor blades and is used to keep the mass center forward of the aerodynamic center. A second lumped mass is included to allow independent choice of mass and mass-moment. The constraints of this construction are clear and are listed on the figure.

In addition, there are minimum constraints on b , h , s , t to hold centrifugal loads and to remain within manufacturable limits. For example a simple minimum

GENERIC BLADE SECTION CONSTRAINTS



Given: c, h
Variable: a, b, d, s, t

Constraints:	Densities:
$0 \leq a \leq h$	τ - Tip Weight & Lumped Mass $\left\{ \begin{array}{l} \text{kg/m}^3 \end{array} \right.$
$0 \leq b \leq c/2 - h$	β - Box Beam
$0 \leq d \leq b - 2s$	η - Honeycomb
$0 \leq s \leq b/2$	σ - Skin (kg/m)
$0 \leq t \leq h/2$	

Figure 1. Typical Blade Cross-Section

constraint on area could come from the centrifugal constraint (not considering bending stress). Thus, if σ_m is the maximum stress and if f is a safety factor then

$$(\sigma_m) \geq f \left[\sum_{i=j+1}^n (M_i) \Omega^2 r_i \right] / A_j$$

$$A_j \geq \frac{f \left[\sum_{i=j+1}^n M_i \Omega^2 r_i \right]}{(\sigma_m)}$$

Of course, when we enter the vibratory-response phase of the work (discussed later) bending stresses will be included.

Similarly, our early work (which neglects flutter boundaries) can nevertheless include flutter criteria in a simplified manner. First, we can choose frequency placement such that no coalescence occurs between flap-lag, flap-torsion, or lag-torsion. Second, we can constrain the five parameters in Figure 1 such that the mass center is always forward of the 1/4-chord, a common design practice to prevent torsion-flutter in rotor blades.

For specific examples, a smaller space of design variables may be used. For example, in the general case one could force the elastic axis to the quarter chord by choosing $b = C/2 - 2h$; or one could choose to allow no lumped mass in the blade interior ($d = 0$). This would reduce the number of design variables from 5 to 4 or 3. If one considers flapping deflection only, then all variables except t and d may be fixed with no loss of generality. (This is equivalent to simply using EI and mass as variables.) Similarly, for inplane one can consider only s and d ; and for torsion, one can consider only b and a . Thus, for each of the three uncoupled cases one has three possibilities: 1) vary stiffness only, 2) vary inertia only, 3) vary stiffness and inertia.

2.2 Definition of Tasks

The work, as proposed here, consists of four primary phases:

1) solution of the inverse problem - given a set of frequencies find the mass and stiffness distribution; 2) solution for optimum forced response - given a loading condition, minimize blade stresses and hub reactions, subject to weight restrictions; 3) improvements to include flap-lag-torsion coupling; and 4) application to existing rotor blades - given an existing blade, improve its dynamic characteristics by realistic structural modification.

These phases are described in detail below. In each phase, work can be performed with nondimensional equations.* Nondimensional quantities to be considered in optimization are ρ (mass distribution), η_y (flapping stiffness), η_z (inplane stiffness), η_x (torsional stiffness), and ϵ (c.g. - e.a. offset). The nondimensional parameters to be included as relatively fixed are γ (Lock number), θ (twist), κ (stiffness of control system), δ (radius of gyration about c.g.) and ξ (aspect ratio).

Aerodynamics assumptions and other constraints are described in the individual tasks indicated below.

2.3 Finite-Element Model

Although tapered, twisted elements are within our capabilities, we introduce here a simpler case which is also of value. The stiffnesses GJ , EI_{zz} , EI_{yy} are assumed to be constant along the length of the element. The mass of the element is assumed to be evenly distributed on the two nodes.

Let the deflection of an element in the y and z directions at a distance x be denoted as $w(x)$ and $v(x)$, for which the displacement models are assumed to be polynomials of third degree. The expressions are given as

* Dimensional cases will also be considered, however, to keep insight.

$$\begin{aligned}
 w(x) &= \frac{v_1}{l^3}(2x^3 - 3lx^2 + l^3) + \frac{v_6}{l^3}(3lx^2 - 2x^3) \\
 &\quad - \frac{v_3}{l^2}(x^3 - 2lx^2 + l^2x) - \frac{v_8}{l^2}(x^3 - lx^2) \\
 w(x) &= \frac{v_2}{l^3}(2x^3 - 3lx^2 + l^3) + \frac{v_7}{l^3}(3lx^2 - 2x^3) \\
 &\quad - \frac{v_4}{l^2}(x^3 - 2lx^2 + l^2x) - \frac{v_9}{l^2}(x^3 - lx^2)
 \end{aligned}$$

where v_1, v_3, v_6 and v_8 represent the bending degrees of freedom in the zx plane and u_2, u_7, u_4 and u_9 represent the bending degrees of freedom in the yx plane

(i) The strain energy due to bending deformation can be expressed as

$$v = \int_0^l \left[\frac{EI_{yy}}{2} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + EI_{yz} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 v}{\partial x^2} + \frac{EI_{zz}}{2} \left(\frac{\partial^2 v}{\partial x^2} \right)^2 \right] dx$$

(ii) The potential energy in tension from the centrifugal force field, which is equivalent to the negative of kinetic energy due to radial displacement, is given by

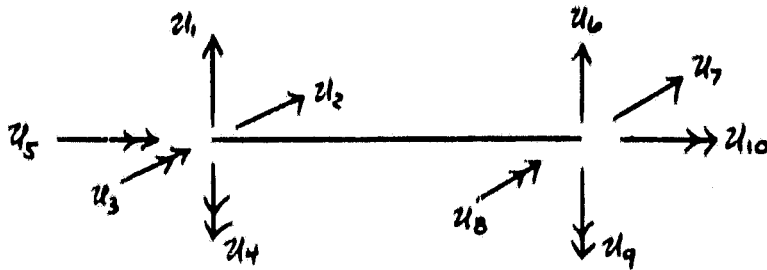
$$-T_K = u = \int_0^l \frac{T}{2} \left[\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 \right] dx$$

where T , tension force, is assumed to be constant along each element.

(iii) The kinetic energy due to inplane displacement is given by

$$T_K = -1/2 \int_0^l m v^2 \Omega^2 dx$$

which is equivalent to a $u = -T$



Meanwhile, the pretwist angle $\phi(x)$, and the torsional deformation $\theta(x)$ are assumed to be polynomials of first degree, and can be expressed as

$$\phi(x) = \phi_1(1 - \frac{x}{l}) + \phi_2(\frac{x}{l})$$

$$\theta(x) = v_5(1 - \frac{x}{l}) + v_{10}(\frac{x}{l})$$

where ϕ_1, ϕ_2 represent the pretwist angle at node 1 and 2, and v_5, v_{10} represent the elastic torsional degree of freedom at each end.

(iv) The torsional energy, due to elastic deformations and centrifugal terms, can be expressed as

$$u = \int_0^l \{ \frac{1}{2} k a_1^2 \frac{T}{A} (\phi' + \theta')^2 + \frac{1}{2} k a_2^2 \frac{T}{A} (\phi' + \theta')^2 + \frac{1}{2} G J \theta'^2 \} dx$$

$$\text{where } k a_1^2 = \iint z^2 dy dz = I_{yy}$$

$$k a_2^2 = \iint y^2 dy dz = I_{zz}$$

(v) The "torsion-rotation" energy under the effect of rotation is given by

$$T_K = - \int_0^l \frac{\Omega^2}{2} (k m_1^2 - k m_2^2) (\phi + \theta)^2 dx$$

where $k m_1^2, k m_2^2$ are mass moment of inertia which can be expressed as

$$k m_1^2 = \iint \rho z^2 dy dz = \rho I_{yy}$$

$$k m_2^2 = \iint \rho y^2 dy dz = \rho I_{zz}$$

Total displacement energy now can be used to form the stiffness matrix from

$$\bar{U} = \frac{1}{2} u^T [K] u$$

where u is the vector of nodal displacements, in the order as $u_1, u_6, u_3, u_8, u_2, u_7, u_4, u_9, u_5, u_{10}$; $[K]$ is the elemental stiffness matrix of order 10.

(vi) The mass matrix will be obtained by the kinetic energy of an element, which is given by

$$T_K = \int_0^L \left\{ \frac{\bar{m}}{2} \left(\frac{\partial v}{\partial t} \right)^2 + \frac{\bar{m}}{2} \left(\frac{\partial w}{\partial t} \right)^2 + \frac{\rho}{2} I_{22} \left(\frac{\partial^2 v}{\partial t \partial x} \right)^2 + \frac{\rho}{2} I_{yy} \left(\frac{\partial^2 w}{\partial t \partial x} \right)^2 + \frac{1}{2} (km_1^2 + km_2^2) \theta'^2 \right\} dx$$

Written in matrix form, the kinetic energy can be expressed as

$$\bar{T} = \frac{1}{2} u^T [m] u$$

where $[m]$ is the mass matrix.

3. STATEMENT OF WORK

3.1 Phase 1 - The Inverse Problem

The first phase of the research effort will involve solution of an inverse problem - that is, find the system structure given the eigenvalues. The solution to the inverse problem is difficult from a mathematical point of view. The inverse problem that will be treated in this phase will be done in successive steps. First, a nonrotating cantilever beam will be used to study bending frequencies. Flapping will be considered in the first beginning. Then, in-plane and torsion will be added. No twist or offset will be included so that the two bending frequencies and the torsion frequency will be uncoupled. A conventional optimal design formulation of one of the problems might be

(1) minimize total weight

$$w = \sum_{i=1}^n \mu_i \Delta r_i$$

With respect to design variables μ and η , subject to constraints

$$\sum r_i^2 \mu_i \Delta r_i = 1$$

$$\Omega_i - \epsilon < w_i < \Omega_i + \epsilon$$

$$\mu_i > \mu_{\min}$$

$$\eta_{\min} > \eta > \eta_{\max}$$

$$\text{and } w_i^2 = \frac{x_i^T K x_i}{x_i^T m x_i}$$

where K and m are structural stiffness and mass matrices with the rigid degrees of freedom removed, x is the eigenvector corresponding to the natural frequency, and w_i is the prescribed natural frequency.

The solution of the above optimization procedure is effected by the optimization program, CONMIN. The efficiency of that program is greatly improved when the derivative of the constraints (with respect to the parameters) is known. Thus, we must consider the derivative of frequency, with respect to the parameters:

$$\frac{\partial(w_j^2)}{\partial\mu_1} = \frac{x_j^T \left[\frac{\partial\kappa}{\partial\mu_1}\right] x_j - x_j^T \left[\frac{\partial m}{\partial\mu_1}\right] x_j}{x_j^T m x_j}.$$

3.2 Phase 2 - Forced Response

In phase two, we will solve for blade structures that minimize forced response. In this step a very simplified model will be used in that we will simply apply a given forcing function. We will not take into account that the blade motions themselves may affect this loading, except that aerodynamic damping will be included. We will assume a very simple lift distribution, such as $u = r^2 \sqrt{1 - r^2}$, that oscillates with integer-multiple frequencies

$$\text{Lift} = \sum_{n=1}^{10} w_n u(r) e^{in\psi}$$

We will then formulate the problem as:

4) Minimize $(e |\text{root shear}| + |\text{root moment}|)$; that is, minimize

$$\sum_{i=1}^n (w_i u_i - \ddot{v}_1 \mu_i)_{\max} \Delta r_i (1 + e r_i)$$

given constraints $\sigma_{\max} < \sigma_{\text{critical}}$,

$$\sum_{i=1}^n \mu_i r_i^2 \Delta r_i = 1, \quad \mu_i > \mu_{\min}$$

The forcing functions will be weighted probably as $w_i = (.3)^i$ or some similar decreasing weightings. The σ will include tension stress plus the oscillatory

stress. One of the major purposes of this will be to decide if minimizing hub loads is equivalent to spacing frequencies half-way between integers. If this is so, then future optimizations would get by without a complicated loads calculation. There is some reason to suspect that this is so. Conceptually, the hub loads could be expressed as

$$L = \sum_{i,j} \frac{w_{ij}}{1 - w_{ij}^2/j^2}$$

Thus, minimizing loads is similar to maximizing $(1-w^2)(1-w^2/4) \dots (1-w^2/n^2)$. The simple factor $(1-w^2)(1-w^2/4)$ has a maximum at $w = 1.58$ and the factor $(1-w^2/4)(1-w^2/9)$ has a maximum at $w = 2.56$. Thus, at (or slightly above) these points may well be optimum.

3.3 Phase 3 - Improvements to Model

In phase three, two parallel efforts are planned. In the first one, the blade structural model will be expanded so as to include twist and c.g. offset. These will couple flap, inplane, and torsion. Therefore, uncoupled analyses will no longer be adequate. We already have computer programs that do this (Ref. 10) but the coupling will mean a more difficult convergence task for the optimization programs. In this phase, c.g. offset will be included as a variable parameter. This implies that "no flutter" must be an added constraint (Ref. 13).

A parallel effort in Phase 3 will be the improvement of the aerodynamic model to include aerodynamic coupling between flap, lag, and torsion. The extent of this work will be partially determined by the results of Phase 2.

Phase 4 - Application to Existing Designs

Finally, these two parallel improvements (structural coupling and aerodynamic coupling) will be combined in a general program. Phase 4 will be to apply this

program to existing rotor-blade designs to determine what structural modifications would be recommended. The fact that our programs already agree well with existing results constitutes a strong starting point for structural improvements.

4. ILLUSTRATIVE EXAMPLES OF STRUCTURAL OPTIMIZATION

4.1 Cantilever Beams with Given Frequency

Some simple examples will be examined and discussed before the utilization of the program CONMIN. In each case the results will be compared with those obtained by previous researchers, if it is available.

The first limiting example is the problem of determining the optimal design of an elastic cantilever beam, such that with a specific natural frequency, the weight of the structure attains the minimum value.

We start with a uniform beam, modeled by ten elements, with a given length of 10 inches, $E = 1.0 \text{ lb-in}^2$, $EI = 10 \text{ lb-in}$, density = 0.042 lb/in^3 , and a specified first lowest natural frequency = 0.6489 rad/sec . We obtain the final stiffness profile shown in Figure 2. Figure 2 is the present result with ten elements.

4.2 Cantilever Beam with Given Weight

A related problem has also been treated by Niels Olhoff [11]. He seeks the design of a cantilever beam that yields a maximum value of a particular higher natural frequency w_n (i.e., of specified order, n) with the volume and length of the beam specified. His work is the dual problem of the example shown in Figure 2. Optimization with respect to the fundamental frequency under the constraint of volume is similar to the one of minimizing weight (or volume) under the constraint of specified natural frequency.

Figure 3 gives the profile of the optimal cantilever for $n = 1$ by Olhoff. One can see that the shapes in Figures 2 and 3 are very similar.

4.3 Cantilever with Tip Mass

Another example problem is to minimize the weight of a cantilever carrying a mass at the tip, subject to the constraint that the fundamental natural

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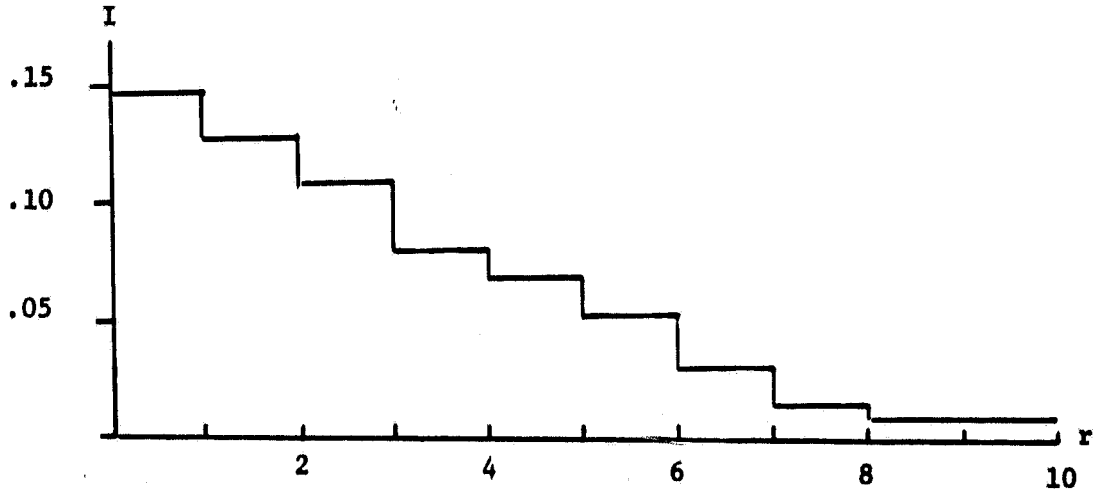


Figure 2. Area Moment of Inertia for Optimum Beam in Present Work.

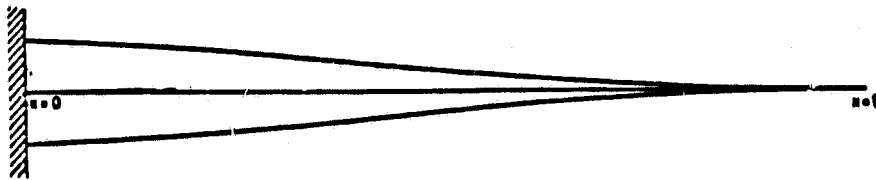


Figure 3. Area Moment of Inertia for Optimum Beam from Reference 11.

frequency must be greater than or equal to a specified value. The problem was originally formulated by Turner [12].

Kahn and Willmert [14] used an optimality criterion method to solve Turner's problem. In this example, four finite elements are used, with the areas of each as the design variables, as illustrated in Figure 4. The specified natural frequency is 17.752 rad/sec. The other initial data are

Modulus of elasticity	= 10.3×10^6 psi
Mass density	= 2.5×10^{-4} lb-s ² /in ⁴
Radius of gyration (A_1)	= 2.0
Radius of gyration (A_2)	= 1.5
Radius of gyration (A_3)	= 1
Radius of gyration (A_4)	= 0.5
Concentrated mass	= 1 lb-s ² /in
Length of each element	= 60 in

where $I = A (\text{radius of gyration})^2$.

The results of the optimization are shown in Table 1. The feasible starting design is described by $A_1 = 200$, $A_2 = 150$, $A_3 = 60$ and $A_4 = 35$.

Table 1 Comparison of Cantilever Beam with Concentrated Mass			
	Ref. [12]	Ref. [13]	This Paper
Iteration	-	23	10
A_1	136.81	136.63	134.60
A_2	118.73	118.7	116.58
A_3	83.591	83.586	82.734
A_4	34.427	34.608	34.898
Weight	2243.0	2242.9	2214.41

It can be seen that excellent results have been obtained using the present CONMIN optimization program.

5. PRELIMINARY CALCULATION FOR ROTORS

5.1 Wind Turbine Blade

The next example is the optimization of a wind turbine rotor blade at 30 rpm. A ten-element model is used. Only the flapping is considered. The area moment of inertia, I , and the lumped weight of each element are taken as the design variables. Young's Modulus, $E = 0.2 \times 10^8$ lb-in, and density = 0.0334 lb/in³ are assumed to be constant. Blade radius, $R = 750$ inches.

Table 2 shows the profile of moment of inertia and the distribution of added weight for the initial and final configurations.

The final profile of the area moment of inertia along the blade is similar as the one in the previous example. The lumped mass is concentrated at the tip of the blade as might be expected given a minimization of weight with a fixed moment of inertia. That is, the moment of inertia remains at the minimum value, as expected. We also note that most of the lumped mass is removed so that only mass necessary to the stiffness elements (or necessary for the autorotation constraint) is maintained.

5.2 Other Considerations

An important aspect of the optimization problem is the existence (or lack of it) of a feasible solution. A "feasible solution" is defined as any set of design variables that satisfy the constraints (whether or not that particular solution is an optimum). It is possible that, if the problem is poorly formulated, that no feasible solution exists. What is more often the case, however, is that there are feasible solutions but that the optimization scheme may not be able to find them. Thus, it is advantageous to have a feasible initial guess so that one is assured that at least a local optimum is possible.

For example, Table 2 illustrates that the first guess is feasible ($w_1 > 2.82$ per/rev). Here we found that CONMIN was able to move from this

Table 2. Optimization of Wind Turbine Blade

FIRST NATURAL FREQUENCY		1.65	1.85 Hz	ROTATING SPEED: 30 RPM		Young's Modulus: $0.2 \times 10^6 \text{ lb-in}^2$		Density: $0.86 \times 10^{-4} \text{ slugs/in}^3$	
SECOND NATURAL FREQUENCY		2.82	3.17 Per/Rev			Data			
NATURAL FREQUENCY		4.85	5.05 Hz						
MASS MOMENT OF INERTIA		8.31	8.65 Per/Rev						
		$\geq 2.5 \times 10^8$	lb-in^2						

first guess through the space of feasible solutions. In other cases, however, when the first guess is not feasible we have found that CONMIN is not able to reach a solution. In such cases, one must add or remove some weight (or add or remove some EI) from the first guess to move into the feasible space. From there, optimization is obtained.

For example, Table 3 presents data for the same wind turbine as in Table 2, but the constraint on the first natural frequency has been lowered to remove it from the dangerous 3/rev range. This implies that the first guess in Table 2 is no longer feasible. In order to overcome this, a lumped mass is added to station 9 (225.4 vs 49.50). This lowers w_1 below 2.621 rev but also lowers w_2 to 0.25/rev. This could be alleviated in one of two ways: 1) move the mass to the node of the second mode, or 2) simply widen the w_2 window. We have done the latter. It is interesting that the added weight is ultimately rearranged to other places and other weight removed such that the new design is no heavier than the optimum in Table 2. Furthermore, w_2 is raised to 8.57 so that the "widened window" had no effect on the solution.

5.3 Helicopter Blade

The design and analysis of a helicopter blade is discussed. Similar to Section 3, only flapping is considered and a ten-element model is used. Density is constant along the blade and equal to 0.18×10^{-3} slugs/in³. Young's Modulus is equal to 0.49×10^7 lb-in² at the root equal to 0.585×10 lb-in² elsewhere. Blade radius is equal to 193 inches. Results are given in Table 4 and 5.

In Table 4, w_1 is in the desired range but w_2 is too small. Furthermore, the autorotational inertia is larger than necessary. In this case, the CONMIN program is able to remove mass and stiffness in such a way to raise w_2 and lower w_1 . The minimum I set as a constraint (0.4) is reached at every point

Table 3. Wind Turbine with Added Mass

Table 3. Wind turbine with rotor 1250											
FIRST NATURAL FREQUENCY		1.4	1.53 Hz	ROTATING SPEED: 30 RPM							
NATURAL : FROM TO		2.4	2.62 Per/Rev	YOUNG'S MODULUS $0.2 \times 10^8 \text{ lb-in}^2$							
SECOND NATURAL FREQUENCY		4.8	5.0 Hz	$0.8673 \times 10^{-4} \text{ slugs/in}^3$							
MASS MOMENT OF INERTIA		$> 2.5 \times 10^8 \text{ lb-in}^2$			Density						
					$a = 0 \quad b = 4.0 \quad s = 0.1$						
					$\eta = 0 \quad h = 2.5 \quad t = \text{variable}$						
					$c = 0$						
ELEMENT	NO	1	2	3	4	5	6	7	8	9	10
LENGTH	in	30	30	75	75	75	75	75	75	75	75
MOMENT OF INERTIA	in^4	I 1012.5	620.0	460.0	302.5	195.0	117.5	55.00	17.50	10.25	10.5
		F 325.6	317.9	302.3	275.7	238.8	192.0	136.6	22.45	10.0	10.0
AREA	in^2	I 65.88	57.4	52.8	47.2	42.0	36.89	30.51	22.86	19.56	19.715
		F 48.15	47.84	47.20	46.05	44.33	41.86	38.34	24.40	19.39	19.39
DISTRIBUTED MASS	lbs	I 65.61	57.17	131.64	117.5	104.66	91.8	75.9	56.93	48.69	49.08
		F 47.95	47.64	117.5	114.6	110.38	104.2	95.46	60.75	48.29	48.29
LUMPED MASS	lbs	I 66.0	57.5	13.25	118.5	105.5	92.5	76.5	57.0	225.4	49.5
		F 0	0	0	0	0	0	0	5.77	0	228.9
		OBJ(1b)	ω_1 Hz	ω_1/Ω	ω_2 Hz	ω_2/Ω	Moment of Inertia				
INITIAL		17873	1.48	2.55	4.81	8.25	3.21×10^8				
FINAL		10371.3	1.48	2.55	5.00	8.57	2.5×10^8				

except the root. The root remained high to keep $w_1 \geq 1.05$. The new blade is one-third the original mass. In Table 5, a stiffer initial guess is used and the frequency w_1 is restrained from significant decrease. Furthermore, w_2 is near its maximum value. In this case, the program CONMIN would like to decrease EI and m, but any removal of material could lower w_1 beyond its lower bound of 1.24/rev. To counter this, the optimization scheme adds EI near the root (to maintain $w_1 > 1.24/\text{rev}$). Furthermore, the lumped mass necessary to maintain autorotational constraint is moved slightly inboard to have less effect on w_1 (keep it high) but more effect on w_2 (keep it low).

Table 5. Helicopter Blade with Stiff Flapping

FIRST NATURAL FREQUENCY		8.8	10.8 Hz	ROTATING SPEED: 425 RPM		YOUNG'S MODULUS: Element 1: 4.9×10^7					
		1.24	1.52 Per/Rev			Rest: 5.85×10^7					
SECOND NATURAL FREQUENCY		17.7	19.7 Hz			Density					
		2.49	2.78 Per/Rev			$a = 0$ $b = 3.75$ $s = .1$ $\eta = 0$ $h = 2.5$ $t = \text{variable}$					
MASS MOMENT OF INERTIA		$\geq 8.7 \times 10^5 \text{ lb-in}^2$									
ELEMENT	NO	1	2	3	4	5	6	7	8	9	10
LENGTH	in	5	10	20	15	15	25	30	35	30	13
MOMENT OF INERTIA	in^4	I 14.28	13.42	20.0	25.8	5.28	0.512	0.406	0.406	0.406	0.406
	F	35.73	22.14	9.198	3.176	1.747	0.494	0.4	0.4	0.4	0.4
AREA	in^2	I 20.6	20.26	22.55	24.05	13.19	0.664	0.594	0.594	0.594	0.594
	F	26.08	23.14	17.32	3.00	1.576	0.652	0.59	0.59	0.59	0.59
DISTRIBUTED MASS	lbs	I 7.16	14.08	31.34	25.07	13.74	1.153	1.238	1.44	1.23	0.536
	F	9.06	16.08	24.06	3.13	1.64	1.132	1.23	1.435	1.23	0.53
LUMPED MASS	lbs	I 0	3.04	1.67	0	6.4	7.46	10.78	4.0	4.55	6.6
	F	0	0	0	0	0	0	28.1	10.15	7.04	0
OBJ		w_1 Hz	w_1/Ω	w_2	w_2/Ω	Mass Moment of Inertia					
INITIAL		141.76	8.883	1.25	10.55	2.76	8.88×10^5				
FINAL		104.86	8.77	1.24	19.7	2.78	8.71×10^5				

6. SUMMARY

Thus far we have completed the following objectives:

- 1) Define the optimization problem and obtain physically reasonable constraints.
- 2) Make CONMIN operational on our own computers and successfully apply it to beam problems with known solutions.
- 3) Apply optimization procedures to typical rotor configurations for flapping.

What remains in the first year is

- 1) Try many more flapping optimization runs to determine the effects of mass versus EI variations, to study the limits on range of frequency placement, to study the limit on number of frequencies placed, to determine to what extent a feasible first guess is necessary.
- 2) Determine the feasibility of using frequency-placement as an objective function rather than as a constraint
- 3) Try inplane and torsion optimization.

After this, we intend to follow the statement of work as planned.

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8. NOTATION

\bar{a}	slope of lift curve, rad^{-1}
a	size of tip mass, m
b	width of box beam, m
c	blade chord, m
d	width of lumped mass, m
e	weighting function
f	safety factor
h	height of box beam, m
μ	nondimensional mass/unit length, $\bar{m} R/m$
μ_x	$GJ/m^2 R^3$
μ_y	$EI_y/m^2 R^3$
μ_z	$EI_z/m^2 R^3$
γ	lock number, $\rho \bar{a} c R^4/I$
K	control system stiffness, $N\text{-}m/\text{rad}$
κ	$K/m^2 R^2$
$\theta(r)$	blade twist, rad
ϵ	nondimensional c.g. elastic-axis offset (offset/ R)
δ	radius of gyration $\sqrt{I_x/I_m R^2}$
ξ	inverse aspect ratio c/r
ρ	density of air
ϵ	small parameter
ψ	Ωt
Ω	rotating speed
σ_m	maximum stress
σ	mass of skin kg/m
τ, β, η	density of lumped mass, box beam, honeycomb kg/m^3

I	necessary inertia for autorotation, kg-m^2
I_x	torsional mass moment of inertia (about c.g.) per unit length, kg-m
EI_y	flapping stiffness, N-m^2
EI_z	inplane stiffness, N-m^2
GJ	torsional stiffness, N-m^2
\bar{m}	mass/unit length, kg/m
m	reference mass, I/R^2 , kg
r	x/R
Δr_1	length of element, m
R	blade radius, m
s, t	thicknesses of box beam, m
$u(r)$	lift distribution, N/m
v_1	displacements, m
w_1	weighting functions
x	length along blade, m